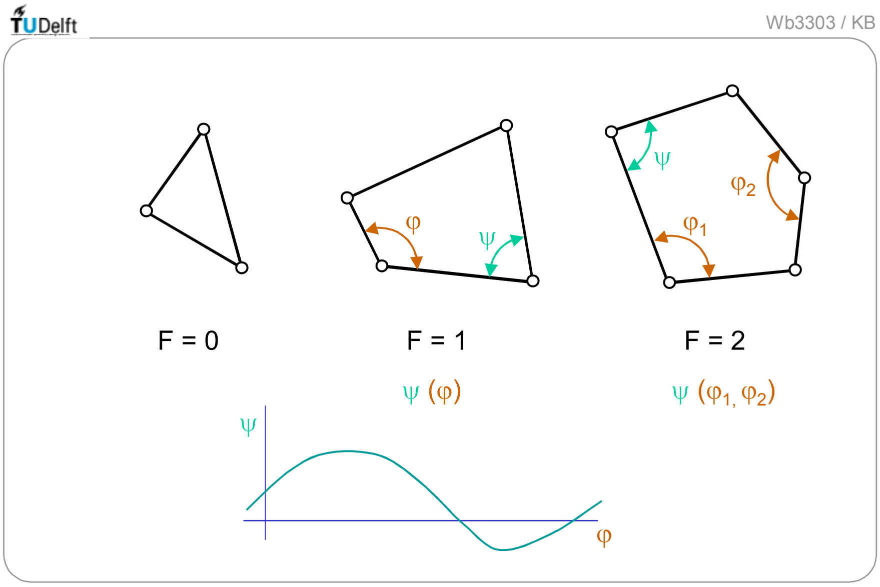

Fig. 1.1.1 Mechanisms and Degree of Freedom F (mobility)
Kinematic transfer function

1 Introduction

The task of a mechanism is to transfer motion and forces between a prime mover and a certain process requiring mechanical energy. Usually this is a subtask of a machine or device, in which the mechanism fulfils a specific function. The design of such machinery is typically a topic in mechanical engineering, especially in production technique (mass production) and transportation technique. In this introduction some terms will be explained to give a global idea of the intention of this course.

1.1 Kinematics and transfer functions

In literature many definitions of the word *mechanism* can be found, like:

1) A system of bodies designed to convert motions of, and forces on, one or several bodies into constrained motion of, and forces on, other bodies (IFToMM [1.1]).
2) Kinematic chain with one of its components (link or joint) connected to the frame.
3) A collection of interconnected rigid bodies that can move relative to one another, consistent with joints that limit relative motion of pairs of bodies (Haug [1.2]).

These definitions show that it is not easy to explain what exactly a mechanism is. It is easier to explain how relative motion between (rigid) bodies can exist. Consider for instance a system of three bars, which form a triangle, see fig. 1.1.1 left. Even when the connections between the bars are parallel hinges (their axes perpendicular to the triangle plane) there is no relative motion possible. A fourth bar added to the chain makes that the system has one possibility to move relatively, that means one geometric quantity (like angle φ in the middle of figure 1.1.1) can be understood as an independently moving variable to which a prime mover should be connected. The relative positions of all other bodies, like angle ψ, depend on this variable. As positions can in general be described with geometric quantities, the *transfer of motion* can be expressed with geometrical functions, which are called (kinematic) *transfer functions*.

The number of possible relative motions of a mechanism is called the *(kinematic) degree of freedom* (DOF) or the *mobility* of the system. In case of a higher mobility each transfer function depends on more variables, see the right part of figure 1.1.1 where five bars are linked by hinges in a chain and the mobility becomes then two.

When all links move in one plane (or in parallel planes) the configuration is called a *planar mechanism* (or co-planar mechanism).

Usually the position of each body is measured relative to one specific body, to which will be referred to further as the *fixed link* or machine frame or ground. The motion of the other links can be considered then as absolute motion.

It depends on the design problem how many transfer functions of a mechanism will be considered. See fig. 1.2.1: the design study of the horizontal movement of a load. The path of the crane tip (point K) can be described with the x- and y- component in a Cartesian co-ordinate system. These individual components are then regarded as transfer functions.

The term *kinematics* refers to the branch of mechanics that deals with the motion of mechanisms, without considering its cause (the forces). Naturally this can only be done assuming rigid bodies. The transfer functions are then the basic information to describe the constrained (rigid) motion. In general a transfer function is non-linear and continuous, so differentiation (with respect to each degree of freedom) is possible. The following chapter 1.2 presents some examples of design studies in which transfer functions are used to express the (desired) motion.

The term *dynamics* refers to the branch of mechanics dealing with the motion and equilibrium of mechanical systems under the action of forces. When mass forces, caused by accelerations, play a role, it is clear that motion needs a description with respect to time. In chapter 1.4 the relation with transfer functions will be explained further.

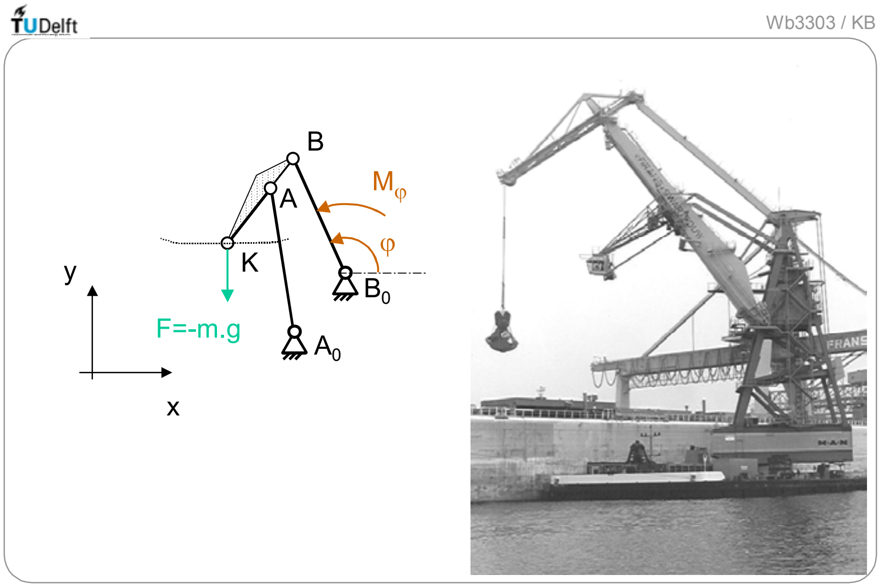

Fig. 1.2.1 Harbour crane (level luffing crane)

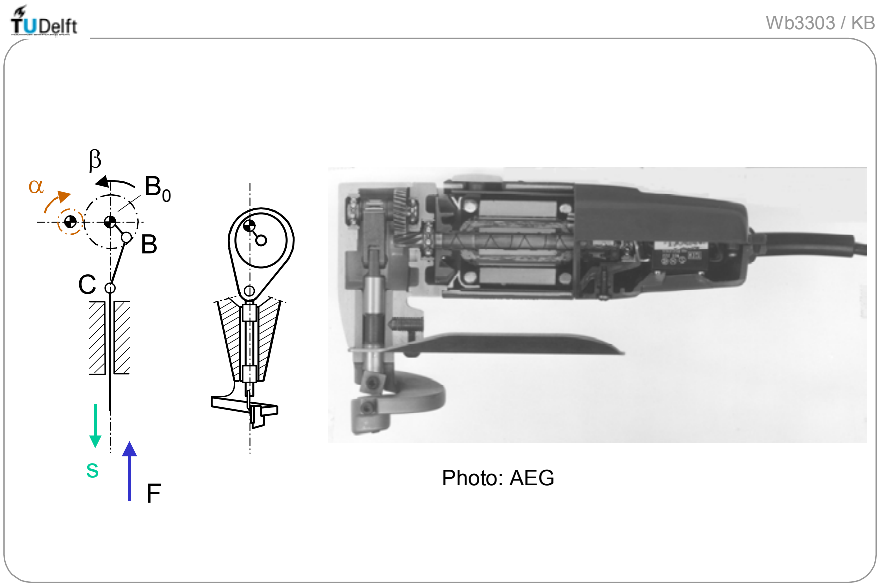

Fig. 1.2.2 Cutting device

1.2 Examples of mechanisms and transfer functions

The intention of this chapter is to recognise existing mechanisms, in particular to reduce it to a kinematic scheme, and to describe its task demonstrating the use of transfer functions.

Harbour crane
Cranes of the type like in figure 1.2.1 (right part) can be found in many harbours. To study the outward motion of the load, the crane can be modelled as a planar four-bar mechanism A_0ABB_0, see fig. 1.2.1 left part. The motion of interest is to be made by point K connected to coupler AB. Point K should move along a straight horizontal line. Cables that are guided across pulleys in point K and for instance B and B_0 hoist the load. Now a horizontal displacement of the load is possible without hoisting and thus without supplying (potential) energy to the load. A relatively light driving motor is then required for the horizontal motion. The design study involves the calculation of the link dimensions such that the path of point K will be horizontal for a long range. This type of mechanism can do this only within certain accuracy.

Considering the (virtual) work of the external forces this can be explained as follows. Suppose that only the load has a mass m and that the prime mover for outward motion drives the angle of link B_0B. Angle φ of link B_0B is thus the degree of freedom. Equilibrium of external forces requires that the total amount of virtual work is zero:

$$M_\varphi \cdot \Delta\varphi - mg \cdot \Delta y_K = 0 \qquad \text{, and thus}$$

$$M_\varphi = mg\frac{\Delta y_K}{\Delta\varphi} = mg\frac{d\,y_K}{d\,\varphi} = mg \cdot y_K^{/} \tag{1.1}$$

Demanding that the driving moment $M_\varphi = 0$ requires thus that the first derivative of the horizontal component y_K (derived with respect to the degree of freedom φ) is zero. This is obviously the same as demanding a horizontal path.

Cutting device
The cutting device depicted in figure 1.2.2 (right part) should manually be moved across a thin plate. The vertically reciprocating knife is shaped such that curved lines can be cut. The mechanism inside consists firstly of a pair of gears to reduce the rotational speed of the driving motor, and secondly of a mechanism to convert the rotation to the reciprocating motion. The latter can be recognised as a slider-crank mechanism (the eccenter B_0B is a constructive realisation of a bar with two hinges at that distance, see figure 1.2.2 left part). Both mechanisms in series, that means the output β of the gear pair is input for the crank, have a total transfer function that is a combined function:

$$s(\beta(\alpha)) \qquad \text{with first derivative} \quad s^{/} = \frac{ds}{d\alpha} = \frac{ds}{d\beta}\cdot\frac{d\beta}{d\alpha} \tag{1.2}$$

The driving torque M_α at the shaft, its rotation angle indicated with α, and the cutting force F_s at the knife, are the external forces that produce no virtual work:

$$M_\alpha \cdot \Delta\alpha - F_s \cdot \Delta s = 0 \qquad \text{which leads to} \qquad M_\alpha = +F_s \cdot \frac{ds}{d\alpha} \tag{1.3}$$

When F_s is required to be high (when the knife starts with cutting) the driving moment should preferably be reduced. This can be done in two ways:

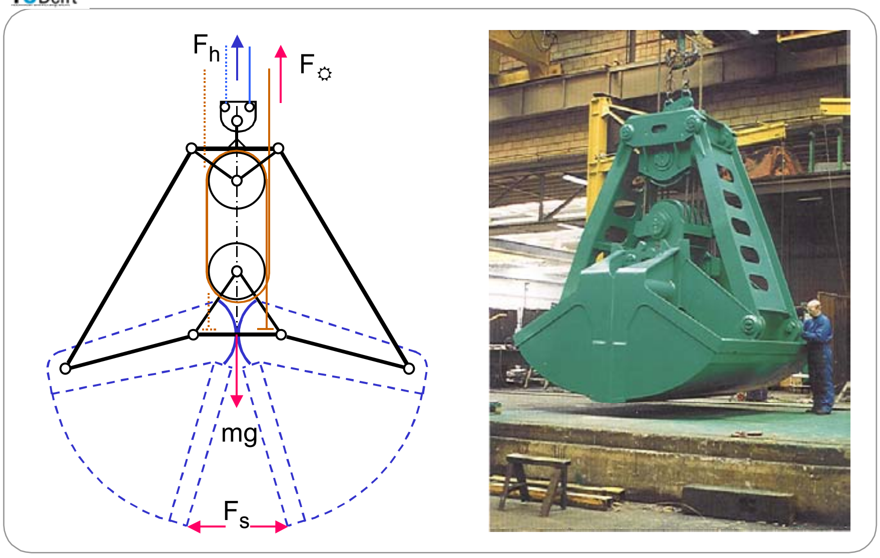

Fig. 1.2.3 Grab (fa. Nemag)

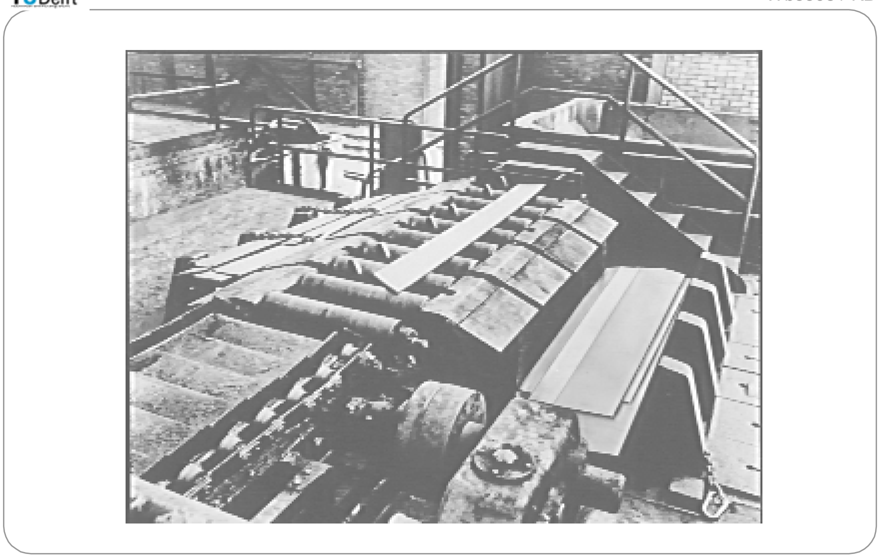

Fig. 1.2.4 Strip stacker (fa. Schloemann)

1. A low value for $d\beta/d\alpha$ (the constant transmission ratio of the pair of gears), and
2. A low value for $ds/d\beta$ during the cutting, this means cutting as close as possible near the lowest position of the knife. The bottom plate, that touches the plate to be cut, can manually be adjusted in height direction of the device housing. The user is supposed to do this for the optimal cutting performance.

Grab for iron ore

Such a grab has usually two (pairs of) ropes: one for suspending the upper sheave block and one for operating the shelves, see figure 1.2.3. Here the situation will be considered when the grab has been placed wide open on the ore and the shelves will be closed by pulling the closing rope. The shelve edges penetrate the ore because the grab (mainly the head block) has a considerable mass m. The penetration depth will be limited by cross beams or plates that will lie down on the ore surface. The design study focuses on the closing force that is maximally available to grab the ore. The external forces involved during the grabbing process are:

- The grab forces F_s working at the opening width s (push back: negative work), and
- The gravity force -mg of the grab, working at its centre of gravity on height h, and
- The vertical force F_ℓ exerted at the closing cable with used length ℓ. Its maximum cannot exceed mg, otherwise the grab will be lifted.

The equilibrium of these forces demands that their virtual work is zero:

$$-F_s \cdot \Delta s + F_\ell \cdot \Delta \ell - mg \cdot \Delta h = 0 \quad \text{from which the closing force can be derived as}$$

$$F_s = F_\ell \frac{d\ell}{ds} - mg\frac{dh}{ds} \qquad , \ F_{s,max} = mg \cdot \left(\frac{d\ell}{ds} - \frac{dh}{ds} \right) \tag{1.4}$$

The closing force will be high if the transfer function $d\ell/ds$ is high. This can easily be achieved with extra pulleys. It is to be noticed that a high value for the transfer function dh/ds (the lift of the grab) may destroy the closing force. The (empty or filled) grab must be self-opening, that means F_s must have a fair value when $F_\ell = 0$. Therefore the transfer function dh/ds must have a certain minimum value.

For operational convenience the head block should preferably be kept on a constant height while digging. During the digging action it seems very natural to consider the head block as the fixed link. Relativ to this head block the motion of the shelves should move then along a straight horizontal path (the digging curve), so that the bottom of the ship can be scraped completely.

Strip stacker

The equipment of figure 1.2.4 is placed at the end of a production line for thin band (steel). Just before the end of the line the band is to be cut into pieces of equal length, which should be collected in a stack at the side. When the stack has reached a certain height, a carrier with a forklift can remove it. Meanwhile the production machinery must continue (rolling mill), therefore a second stacking place has been added at the other side. A mechanism must divide the steel plates either to the right or to the left.

The basic tool is a set of fingers that can move reciprocating between the rollers supporting the band material. These fingers could simply be attached to a rocker with its rotation point D_0 sufficiently below the supporting rollers, see figure 1.2.5. Each plate will be pushed over a border and will fall down at the stack. To avoid collision with the next plate the fingers should sink under the level of the rollers, return to the other side, rise again and push the new plate. Because this vertical motion concerns only a small distance the bearing of the rocker is placed on a slider, which should perform the required vertical motion. So a second mechanism (slider-crank mechanism $B_0B_1D_0$) has been introduced to move this slider. In this way the tip of each finger moves along an ellipse-like path, see point D_2 in figure 1.2.5.

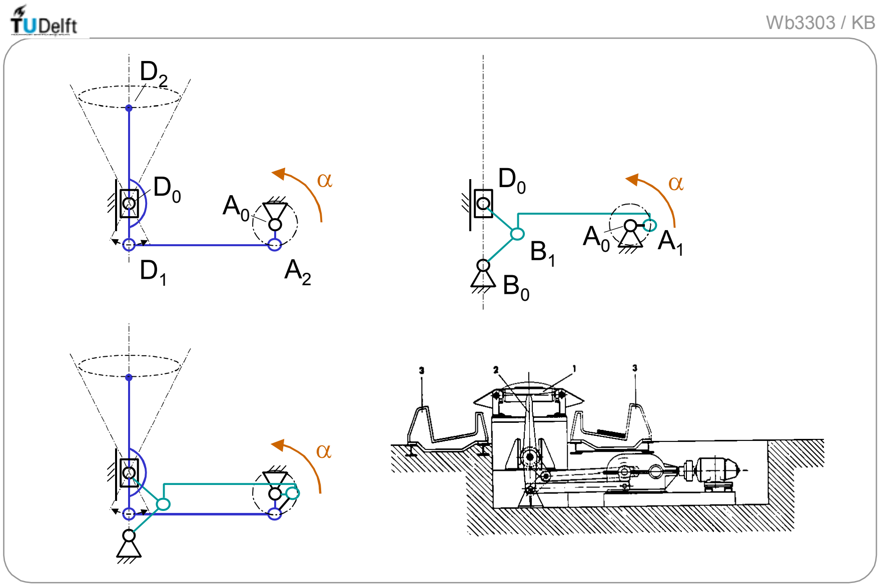

Fig. 1.2.5 Strip stacker mechanism, kinematic scheme

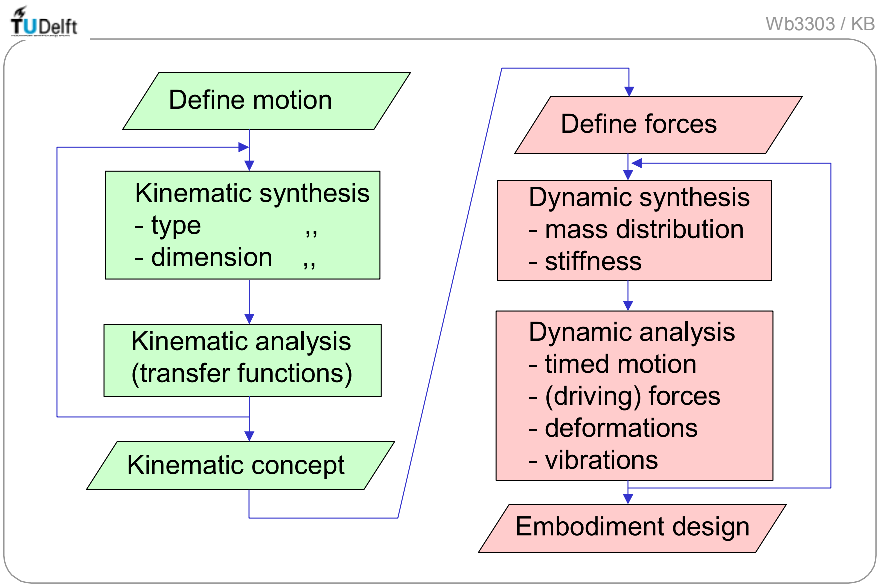

Fig 1.3.1 Kinematics and dynamics in the design process of mechanisms

Both mechanisms must have a crank at the same driving shaft (point A_0), which is in its turn the output shaft of a driving motor and a gearbox. For better inspection and maintenance it is a good idea to locate the driving components aside the machine. So relatively long bars A_1B_1 and A_2D_1 are used to transfer the motion from the driving shaft to the actual machine elements to move.
To push the plates to the other side the driving direction of the motor has to be reversed. The mechanism has a waiting position (point D_2 below) where the motor will be stopped after having a plate pushed. The motion cycle will be started again after a new strip has been detected.

1.3 The design process of mechanisms

A design process starts with a problem definition and will be followed by choosing solution principles and technical realisations. Such a general description is for instance given in [1.3]. Here an interpretation directed to mechanism design will be presented.
The basic idea is, that first the kinematic principle should be determined to solve the motion problem (to be defined with transfer functions). After that the links can be dimensioned for transfer of forces (dynamics), see fig. 1.3.1
A further refinement can be made using the general words *analysis* (examination of an existing system) and *synthesis* (combine elements to create a system).

To design a mechanism usually the following stages have to be passed:

- Motion definition by specifying one or more desired transfer functions.
- Kinematic synthesis, to be divided further into:
  - ⇒ Type synthesis (choice for the mechanism type). Designers prefer simple mechanisms. About the variety of mechanism types more in chapter 2.
  - ⇒ Dimension synthesis (calculation of the kinematic parameters such as link lengths).
- Kinematic analysis. As a result of dimension synthesis the desired transfer functions will be generated by the chosen mechanism. This could be verified. Other design demands like transfer quality (to be explained in chapter 5.5) or space occupation in the machine can be investigated here.
- Conceptual kinematic design. The best or most promising kinematic concept must be chosen.
- Force definition. The forces needed to perform the machine process must be specified.
- Dynamic synthesis. The non-kinematic body dimensions like bar thickness and the material must be chosen. The kinematic model can be extended (for instance: add extra mass to balance forces, specify that a body has flexibility etc.).
- Dynamic analysis. Here all forces and deformations can be calculated to verify that the links are strong and stiff enough. Usually mass forces play a role, so timed motion must be regarded. The driving force is required to select a driving motor. A motor model can be specified to calculate the actual forces.
- Embodiment design. The final shape of the individual bodies is made such that no collision between bodies occurs.

It will be clear that this design process has iterative loops. The most important ones are drawn in the figure 1.3.1.

This book mainly gives attention to the design stages kinematic analysis and dynamic analysis of mechanisms.

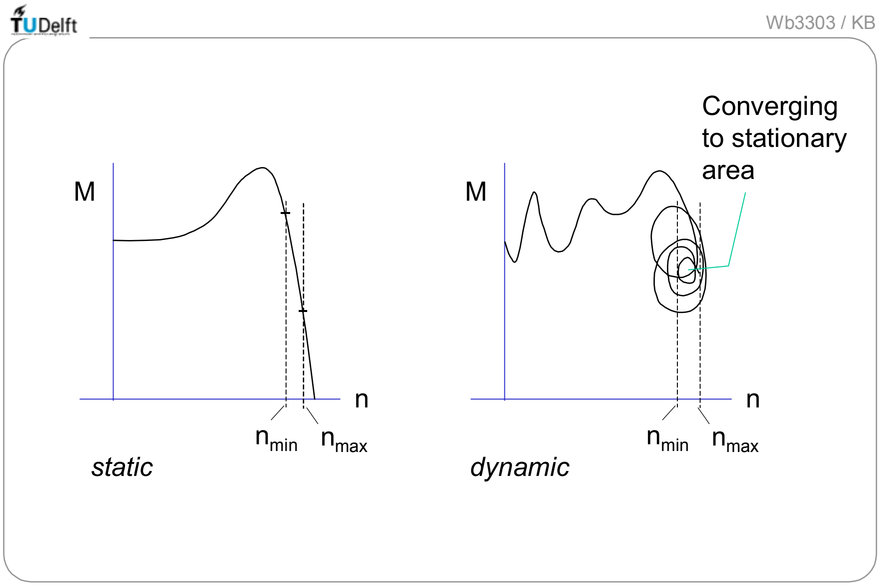

Fig. 1.4.1 Static drive characteristic and typical dynamic behaviour with a mechanism

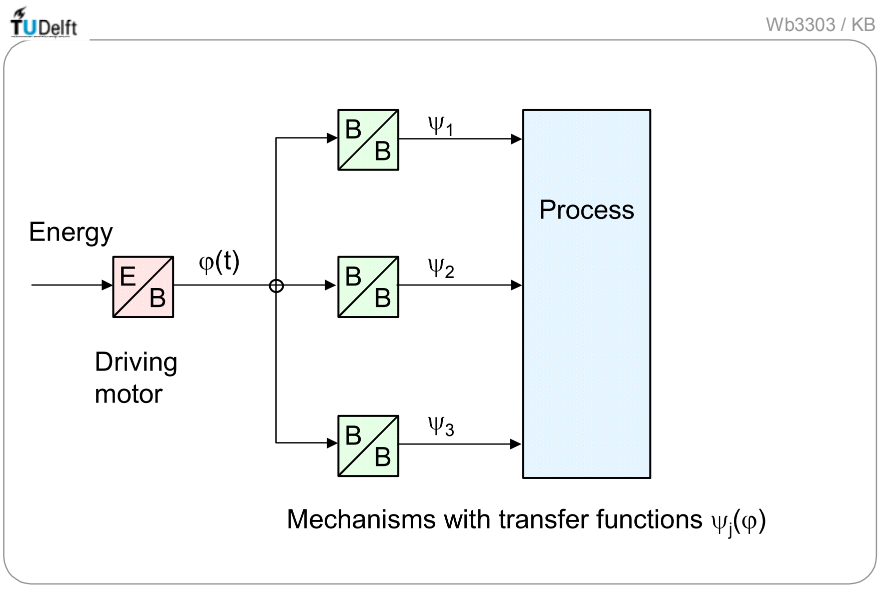

Fig.1.5.1 Machine concept with central driving shaft

1.4 Timed motion

Motion is defined as “changing position of a body relative to a frame or reference” [1.1]. Timed motion species that the changing occurs with respect to time.

As soon as all degrees of freedom of a mechanism are associated with a geometrical quantity, transfer functions of the mechanism are defined. Usually the degrees of freedom will be driven (input motion), which means that they will move with respect to time. Timed motion of any geometrical quantity, here indicated with ψ, dependent on input motion $\varphi(t)$, can be defined then with

Position $\psi(\varphi(t))$ (1.5)

Velocity $\dot{\psi} = \dfrac{d\psi}{dt} = \dfrac{d\psi}{d\varphi} \cdot \dfrac{d\varphi}{dt} = \psi^{/} \cdot \dot{\varphi}$ (1.6)

Acceleration $\ddot{\psi} = \dfrac{d^2\psi}{dt^2} = \dfrac{d^2\psi}{d\varphi^2}\left(\dfrac{d\varphi}{dt}\right)^2 + \dfrac{d\psi}{d\varphi} \cdot \dfrac{d^2\varphi}{dt^2} = \psi^{//}\dot{\varphi}^2 + \psi^{/}\ddot{\varphi}$ (1.7)

Clearly time derivatives and transfer functions are different quantities. Occasionally it harms not much when the wrong term is used. For instance: when an input angle φ rotates with constant speed ($\dot{\varphi}$ = constant and $\ddot{\varphi} = 0$) the velocity and the acceleration of the output motion differ only by a constant factor from the first and second order transfer function. The cutting device of paragraph 1.2 is designed to operate with constant speed. Stating that the driving moment is low when the velocity of the knife is high is qualitively correct. This velocity is however also dependent on the driving speed, but this is not a property of the mechanism. So the relation between driving moment and cutting force can be described more properly with the first order transfer function.
Transfer functions and time derivatives become basicly different in case of non constant input velocity. This is for instance the case when the driving motor starts or stops moving ($\dot{\varphi} = 0$ and $\ddot{\varphi}$ = nonzero). The reader may verify that the acceleration of the output $\ddot{\psi} = \psi^{/} \cdot \ddot{\varphi}$ depends then on the first order transfer function, but has nothing to do with the second order transfer function!
It is certainly the scope of this book to consider nonuniform driving motion (due to the characteristic of the driving motor). Such driving characteristics are well known, see for instance fig. 1.4.1. It will be clear that for such studies of the system “prime mover – mechanism – process” time derivatives and transfer functions need to be used correctly.

1.5 Machine concepts

The system prime mover – mechanism - process can be recognised in many machine concepts. Most concepts can be reduced to, or consist of a combination of, one of the following three basic types:

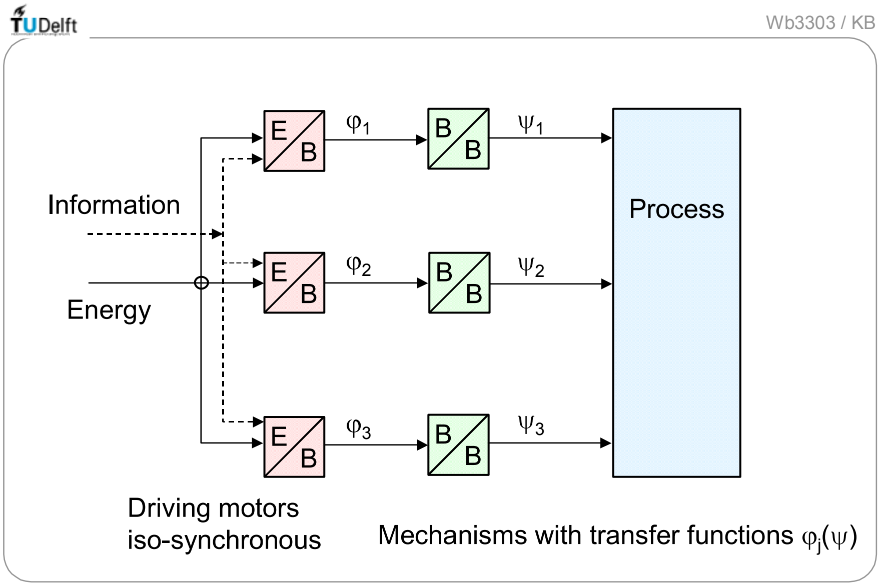

Fig. 1.5.2 Machine concept with distributed driving shafts

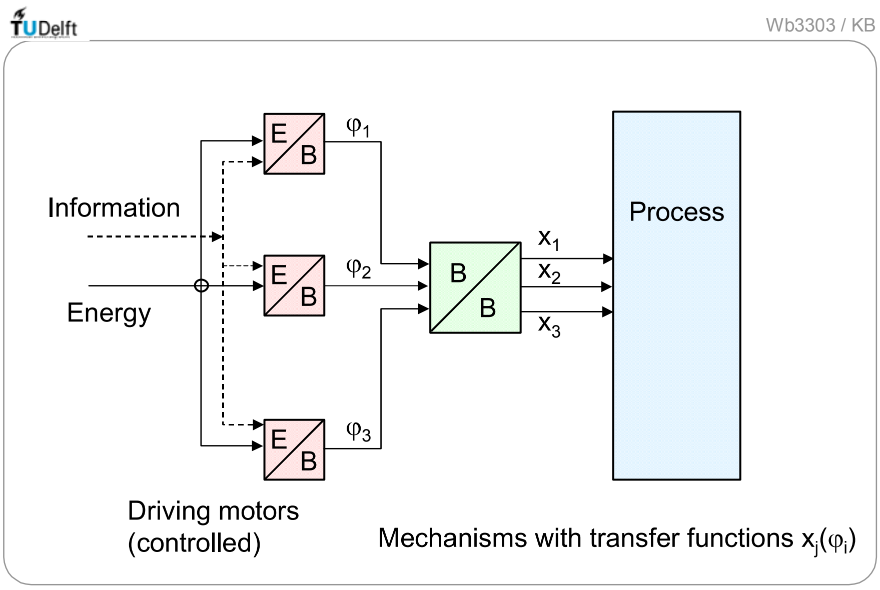

Fig. 1.5.3 Machine concept with multi-degree of freedom mechanism (industrial robot)

- With one drive and a central shaft, see figure 1.5.1. Electrical energy E is transformed to mechanical energy of the shaft by the drive (motion of φ). The mechanisms transfer mechanical energy (motion of ψ_i) defined by kinematic transfer functions according (1.6) and (1.7) to perform the process under action.
- In a more modern variant each mechanism has its own driving motor which runs iso-synchronously (same speed and in-phase) by electronic control, see figure 1.5.2. This variant is often called “electronic shaft”. In a cheap realisation stepping motors can be used that need only one central control.
- The system of figure 1.5.3 has multiple inputs (drives) and outputs. This is typical for an industrial robot. Each drive has now its own control for its motion as a function of time.

The different concepts may need different design actions.
The central shaft concept allows usually that the kinematics part can easily be done without considering time. The driving motor is probably just a big one that drives at approximately constant speed. But when flexibility of links is to be taken into account seriously, then the whole system should be considered as one complex mechanism.
In the electronic shaft concept, where the driving motors should preferably be distributed as close as possible to the action place, the size of these motors may deserve much attention. It is very advantageous then to know the required driving forces precisely. Small stepping motors can be applied then with less risk for loosing steps. Flexibility of links can probably be considered with a separated dynamic model for each mechanism.
In the multi-degree of freedom concept time is the quantity that relates the different drives. Therefore desired motion can only be described as a function of time. Timed motion should be considered then, even in the kinematic design part. Kinematic transfer functions may however still play a role in typical geometric questions, such as the description of the maximum workspace of a robot. In many design cases it concerns a positioning problem including certain position accuracy. Vibration analysis can be important then.

It is certainly the intention of this course to support all those design activities.

1.6 References

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